PROFILE OF A RADIATING DISK OF MINIMUM WEIGHT

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The variational problem of determining the optimum (with respect to weight) profile of a radiating disk is formulated and solved. The results of a computer calculation are presented. Radiating disks of constant thickness, triangular profile and optimum profile are compared in terms of weight.

There have been numerous studies of various radiator elements for dissipating heat under vacuum conditions. Flat fins of various shapes have been thoroughly investigated [1–4], but the same cannot be said of radiating disks. In [5], which is concerned with radiating surfaces, only the premises for the formulation of the problem of a system of radiating disks are given. In [6] the problem of determining the dimensions of a disk of constant thickness and optimum weight is formulated and the data needed for the calculation are presented.

The present paper is a development of [6] and includes the formulation and solution of the variational problem of determining the profile and dimensions that will give the minimum ratio of disk weight to radiated heat. The starting data are: the radius (R) of the internal opening of the disk, the temperature (T_0) at the surface of the internal opening, the quantity of heat (Q) radiated by the disk, the thermal conductivity and density (λ, ρ) of the disk material, and the emissivity (ε) of the radiating surface.

In [6] on the basis of a consideration of the heat balance for an element $x dx d\alpha$ of the disk under conditions of zero ambient temperature (Fig. 1) and the expression for the relative weight of the disk (γ is the ratio of disk weight to radiated heat), it was shown

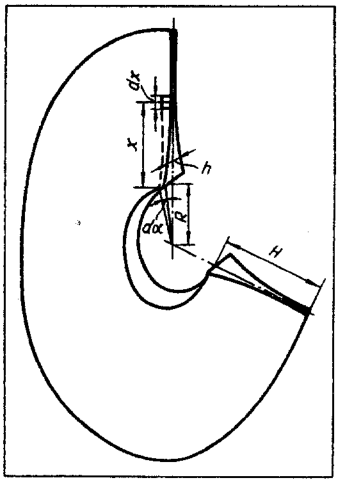

Fig. 1. Disk geometry.

that the relative weight of the radiating disk is a product of two factors, the first of which depends only on the starting data:

$$\gamma = \frac{g\rho (Q/\Pi_0)^2}{\lambda \varepsilon^2 \sigma^2 T_0^9} \; \frac{\int_0^1 \varphi \, dt}{\bar{Q}^3 c}.$$

Here, Π_0 is the radiating perimeter at the base; in our case, $\Pi_0 = 4\pi R$; $\varphi = F_x/F_0$ is the ratio of the cross-sectional area of the disk in section x to the corresponding area at the base; c, the conductivity parameter, is given by the expression

$$c = \frac{\varepsilon\sigma T_0^3 H^2 \Pi_0}{\lambda F_0}, \tag{1}$$

and $\bar{Q}$, the efficiency of the disk, expressed by the formula

$$\bar{Q} = \int_0^1 (1 + kt)\,\theta^4 \, dt, \tag{2}$$

is determined by solving the nonlinear differential equation

$$\frac{d(\varphi \, d\theta/dt)}{dt} = c(1 + kt)\,\theta^4 \tag{3}$$

with the boundary conditions $\theta = 1$ at $t = 0$ and $d\theta/dt = 0$ at $t = 1$.

In expressions (2) and (3),

$$\theta = T/T_0, \; k = H/R \text{ and } t = x/H,$$

where T is the temperature of the disk.

The need to radiate a given quantity of heat imposes the additional condition

$$k\bar{Q} = \frac{Q}{\varepsilon\sigma T_0^4 \Pi_0 R} = q. \tag{4}$$

The quantity q characterizes the heat load on the radiating perimeter of the base of the disk and, in the optimization problem, is given.

Thus, we may conclude that the solution of the problem of determining the profile and dimensions of the disk of minimum weight depends only on the quantity q and reduces to finding the function φ and the parameters c and k that minimize the functional

$$\Phi = \frac{\int_0^1 \varphi \, dt}{\bar{Q}^3 c} \tag{5}$$

with constraints (1)–(3) and (4).

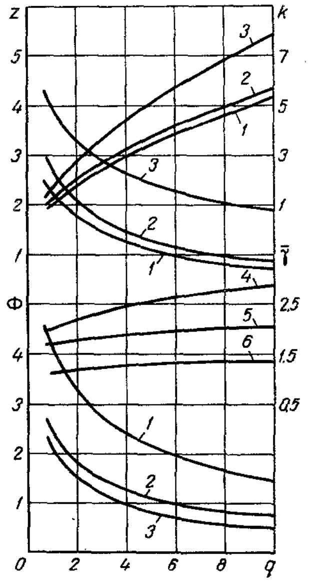

Fig. 2. Geometric and weight characteristics of radiating disks: 1) disks of constant thickness; 2) disks of triangular profile; 3) disks of optimum profile; 4) ratio of the weight of disks of constant thickness to the weight of corresponding disks of optimum profile; 5) ratio of the weight of disks of constant thickness to the weight of disks of triangular profile; 6) ratio of the weight of disks of triangular profile to the weight of disks of optimum profile.

It is not possible to solve this problem analytically. Instead we use one of the direct methods of solving variational problems—the method of gradients [7]. This method requires that the form of the sought extremal be given. In our case, it is convenient to use a function of the following form:

$$\varphi = \frac{a_1^t - a_2 - a_3 t}{1 - a_2},$$

The coefficients a_1, a_2, and a_3 are found from the conditions

$$\varphi_{t=1} = \varphi_k, \quad \varphi'_{t=1} = \varphi'_k \text{ and } \varphi'_{t=0} = \varphi'_0.$$

The maximum of functional (5) was determined on an M-20 computer. Since the numerical machine solution does not admit a value of zero for the function φ, 0.01 was taken, instead of zero, as the minimum value of that function. This does not have any appreciable effect on the optimum profile and dimensions of the disk.

Table 1

Coefficient of the Optimum Profile Function

Coefficient	Value of coefficients for q equal to: 0.75	1.25	2.5	5.0	7.5	10.0
a_1	0.2391	0.3250	0.5375	0.7000	0.9000	0.95
a_2	0.5769	0.6871	0.8699	0.9492	0.9948	0.9987
a_3	−0.3421	−0.3653	−0.3337	−0.2497	−0.0948	−0.0488

The calculations show that, for the range of values of q considered, the optimum profile is characterized by $\varphi'_k = 0$ and $\varphi_k = 0.01$.

Table 1 and Fig. 2 show the results of solving the variational problem in the form of the dependence on q of: the coefficients a_1, a_2, and a_3; the quantity k, characterizing the thickness of the disk at the base; and the quantity Φ, characterizing the relative weight of the disk. The relation between z and the thickness of the disk at the base (h_0) is given by the formula

$$h_0 = 2\,\frac{(Q/\Pi_0)^2}{\lambda\varepsilon\sigma T_0^5}\,z.$$

The variation of the thickness of the disk along the radius is characterized by the relation

$$h/h_0 = \varphi/(1 + kt).$$

The necessary data for constructing the optimum profile are presented in Table 2.

To compare radiating disks of optimum profile with disks of other simpler profiles, we plotted (Fig. 2) curves characterizing the least weight and optimum dimensions of radiating disks of constant thickness and disks of triangular profile. The comparison shows that going over from disks of constant thickness to disks of triangular profile makes it possible to reduce the weight of the disk by a factor of 1.1–1.4 in comparison with a disk of triangular profile, and by a factor of 1.9–2.9 in comparison with disks of constant thickness. In this case, the greater the heat that the disk must radiate, the greater the advantage in weight.

Table 2

Variation of Relative Thickness of Disk of Optimum Profile along the Radius

t	Ratio h/h_0 for q equal to: 0.75	1.25	2.5	5.0	7.5	10.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.6736	0.6466	0.5972	0.5271	0.4848	0.4505
0.2	0.4502	0.4205	0.3711	0.3074	0.2734	0.2468
0.3	0.2961	0.2713	0.2331	0.1861	0.1630	0.1450
0.4	0.1896	0.1714	0.1449	0.1131	0.0983	0.0867
0.5	0.1166	0.1044	0.0874	0.0673	0.0583	0.0511
0.6	0.0676	0.600	0.0500	0.0380	0.0329	0.0288
0.7	0.0360	0.0316	0.0261	0.0197	0.0171	0.0149
0.8	0.0170	0.0147	0.0119	0.0089	0.0076	0.0066
0.9	0.0072	0.0060	0.0046	0.0033	0.0028	0.0024
1.0	0.0042	0.0033	0.0023	0.0016	0.0013	0.0011

REFERENCES

1. G. L. Grodzovskii, Izv. AN SSSR, Energetika i avtomatika, no. 6, 1962.
2. Y. G. Bartas and W. H. Sellers, Trans. ASME, Cer. C., J. of Heat Transfer, 82, no. 1, 1960.
3. D. B. Mackey, Design of Space Power Systems [Russian translation], Izd. Mashinostroenie, 1966.
4. B. A. Solov'ev, IFZh [Journal of Engineering Physics], 14, no. 3, 1968.
5. E. R. G. Eckert, T. F. Irvine, and E. M. Sparrow, ARS Journal, 30, no. 7, 1960.
6. B. A. Solov'ev, Izv. AN SSSR, Energetika i transport, no. 1, 1966.
7. H. J. Kelley, collection: Optimization Techniques with Applications to Aerospace Systems [Russian translation], G. Leitmann, ed., Izd. Nauka, 1965.

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